Tradability versus Performance:
The Role of Liquidity in Minimum Variance Smart Beta Products

Frank Siu
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Minimum Variance Smart  
Beta Products  

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Low-volatility themed strategies have been among the most popular “smart beta” index products introduced in recent years, and minimum variance in particular has become a widely adopted approach to implementing low-volatility exposure. Minimum variance uses a risk model and optimization techniques to systematically construct portfolios that have the lowest possible ex-ante risk. On the basis of simulated backtests, these strategies often boast very impressive track records, both in terms of market outperformance and risk reduction. Can practitioners realistically achieve such advertised performance once transaction costs are taken into account? This question is especially relevant to minimum variance because stocks that trade infrequently may appear low risk and hence be favored by the optimizer and held in large amounts. In the following analysis, we attempt to address the following questions:  

- From a risk perspective, to what extent is the “theoretical” low risk of these strategies driven by illiquidity masquerading as low volatility?  
- Do returns of minimum variance strategies encapsulate some form of liquidity premium in addition to the outperformance of low-risk stocks?  
- If there is a tendency to tilt towards smaller and less-liquid stocks, what can be done to ensure tradability of minimum variance portfolios?  

RISK PROFILE OF MINIMUM VARIANCE STRATEGIES  

To begin, we examine the performance of minimum variance strategies in several key markets around the world. Rather than carry out an unconstrained minimum variance optimization, for meaningful results, we adopt a portfolio strategy with realistic constraints on trading and asset concentration:  

- Minimize total risk as predicted by a factor risk model. We use commercially available equity risk models developed by Axioma.  
- Monthly rebalancing frequency, maximum of 5% turnover (one-way) each time.  
- UCITS compliance in the form of a 5/10/40 rule—no single issuer can exceed 10% weight, and all holdings greater than 5% weight must total less than 40%.  
- The inverse Herfindahl Index of optimal portfolio weights $w$ must be at least 30% that of the benchmark index $b$:  

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Avoid tiny holdings; minimum holding size is 3 bps.

The historical performance of this strategy applied to several markets is shown in Exhibit 1, with generally very strong returns, both in total and risk-adjusted terms. Sharpe ratios in all but one case are well above 1.0.

Exhibit 2 analyzes the risk profile of these strategies by looking at their exposure to certain style factors. Whereas their exposure to fundamental-driven styles like value and growth are relatively small in magnitude and not always consistent across markets, all strategies exhibit large and persistent negative tilts on liquidity, size, and volatility. Our minimum variance portfolios, therefore, are characterized by heavy concentration in small, less-actively traded, low-volatility names. Large positive exposure to the leverage factor is also a common theme, and this can be explained as a by-product of low-volatility strategies’ preference for overweighting utility companies, which often carry a heavy debt load.

MEASURING TRADABILITY

Having established minimum variance strategies’ affinity for small and illiquid stocks, we next analyze the implications for portfolio liquidity. Let us define the days-to-trade for a given position \( i \) as:

\[
\text{days To Trade}_i = \frac{h_i}{c \cdot ADV_i} \quad (1)
\]

This is an estimate of the number of days required to create or liquidate the position \( h_i \) given some estimate of its trading volume, both expressed in units of currency. The constant \( c \) represents the participation rate, for example, assuming only being able to trade 10% of daily volume.

Similarly, we can compute a weighted average to represent a portfolio’s overall liquidity. Summing up Equation (1) across all positions is possible and may even seem more intuitive at first, but fails to capture the fact that trading in multiple stock positions can occur simultaneously, rather than sequentially. Instead, we define a portfolio’s weighted average days-to-trade as:

\[
\text{weighted Day To Trade}_p = \frac{\sum_{i \in p} w_i \cdot \frac{h_i}{c \cdot ADV_i}}{\sum_{i \in p} w_i \cdot c \cdot ADV_i} = \frac{\sum_{i \in p} w_i^2 \cdot N}{\sum_{i \in p} w_i^2 \cdot c \cdot ADV_i} \quad (2)
\]

\( N \) represents the portfolio notional value. In Exhibits 4(a) through 4(g) we compute the days-to-trade for each position in the given portfolio at each point in time,
and organize the portfolio into “liquidity layers” depending on the length of time required to trade each position, assuming a notional value of $500M USD and participation rate of 20% ADV. In Exhibit 3, we compute the weighted average days-to-trade for each portfolio, compared to its corresponding capitalization-weighted broad market benchmark index. The figures may seem somewhat high, partly because they are median values sampled over a ∼13 year period. Market capitalizations and trading activity in many markets were significantly lower during the older periods, so the $500M notional would have corresponded to a tougher liquidity assumption, but the values nonetheless provide a rough indicator of tradability.

At this point, it is evident that our minimum variance portfolios suffer from severe liquidity problems. Even assuming a high participation rate and modest small portfolio size, many portfolios require several months to trade. In the most extreme case (Hong Kong), the minimum variance portfolio would theoretically require over a year to build! Even in the U.S., where there is ample liquidity, the minimum variance strategy takes over three days to trade; an equivalent amount invested in a capitalization-weighted benchmark (e.g., Russell 1000) requires less than half an hour of trading. Furthermore, these liquidity measures are weighted averages of individual asset positions and do not reflect the largest offenders; in Exhibit 4(a) through 4(g) there are almost always positions in the portfolio requiring over 100 days to trade, rendering parts of these portfolios prohibitively costly to implement. In short, our minimum variance portfolios are not tradable. What can be done?

LIQUIDITY VERSUS PERFORMANCE

If the stellar returns in Exhibit 1 are not attainable due to insufficient liquidity, what happens as we gradually improve the portfolios’ tradability? Will more liquid minimum variance portfolios necessarily underperform less liquid ones? In this section, we attempt to illustrate the relationship between liquidity and minimum variance performance (both in terms of return and effectiveness of risk reduction). We run a battery of backtests
EXHIBIT 4
Breakdown of Portfolio into Liquidity Layers, $500M USD Notional, 20% ADV

(a) Australia
(b) Canada
(c) Eurozone
(d) Hong Kong
(e) Japan
(f) United Kingdom
(g) United States
with different parameters aimed at improving tradability, such as:

- Removing illiquid stocks from the universe of eligible stocks, based on their ranking in terms of trading activity. For example, excluding the lowest decile of stocks ranked by ADV.
- Capping the exposure of illiquid stocks at their weight in the corresponding capitalization-weighted index or imposing narrow bounds around the benchmark weights so only small overweights are permitted.
- “Targeting” an explicit weighted average days-to-trade for the aggregate portfolio, or for specific segments thereof, or capping the total days-to-trade for holdings within certain subsets of holdings.
- In all the above, we experiment with different parameters for identifying what constitutes an “illiquid” stock, from static ADV thresholds to more dynamic methods that can evolve with market conditions and benchmark structure.

The results are displayed in Exhibit 6(a) through 6(f). In all but one market, there is a clear positive relationship between illiquidity, as measured by weighted average days-to-trade, and performance, both absolute and risk-adjusted. The relationship is statistically very significant (see Exhibit 5), with rank correlation between weighted average days-to-trade and the various performance metrics in excess of 80%. The U.K. is the sole exception; its characteristics are directionally opposite to all the others in our sample, and could be a promising candidate for a subsequent and more detailed study.

The relationship between liquidity and risk is also very clear, though interestingly the sign of this relationship is not consistent across markets. In Australia, Canada, and the U.K., the more tradable minimum variance portfolios incur lower realized risk, whereas the opposite is seen in the Eurozone, Hong Kong, Japan, and the United States. One could speculate a possible linkage with market structure. In Australia and Canada, where there is less breadth in terms of number of names available in the investable universe, there are fewer opportunities for diversification, and venturing into less-liquid segments of the market runs greater risk of encountering stocks that seldom trade but occasionally experience large return spikes. In markets with greater diversity, there may indeed exist some smaller, less-liquid yet low-risk (and perhaps often-overlooked!) stocks.

**ENFORCING LIQUIDITY**

Many strategies ensure portfolio liquidity by incorporating constraints on individual positions based on the corresponding stock’s trading volume, but this requires specifying a portfolio notional size. This becomes difficult when designing a “generic” strategy that may be

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**EXHIBIT 5**

**Summary Statistics from Regressing Each Performance Metric Against Weighted Average Days-to-Trade**

Beta values are scaled by 1000, absolute t-statistics in brackets.

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Canada</th>
<th>Eurozone</th>
<th>Hong Kong</th>
<th>Japan</th>
<th>U.K.</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Return</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta (t-stat)</td>
<td>0.33 (7.48)</td>
<td>0.4 (10.49)</td>
<td>1.83 (8.7)</td>
<td>0.22 (9.38)</td>
<td>0.61 (9.61)</td>
<td>-2.25 (5.7)</td>
<td>12.61 (20.05)</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.49</td>
<td>0.66</td>
<td>0.57</td>
<td>0.60</td>
<td>0.61</td>
<td>0.36</td>
<td>0.87</td>
</tr>
<tr>
<td>Rank Corr.</td>
<td>0.80</td>
<td>0.89</td>
<td>0.81</td>
<td>0.78</td>
<td>0.80</td>
<td>-0.60</td>
<td>0.94</td>
</tr>
<tr>
<td><strong>Realized Risk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta</td>
<td>0.12 (6.79)</td>
<td>0.05 (8.39)</td>
<td>-0.81 (7.63)</td>
<td>-0.07 (7.13)</td>
<td>-0.65 (19.73)</td>
<td>0.91 (5.2)</td>
<td>-6.58 (18)</td>
</tr>
<tr>
<td>r2</td>
<td>0.44</td>
<td>0.55</td>
<td>0.50</td>
<td>0.47</td>
<td>0.87</td>
<td>0.32</td>
<td>0.85</td>
</tr>
<tr>
<td>Rank Corr.</td>
<td>0.75</td>
<td>0.70</td>
<td>-0.77</td>
<td>-0.72</td>
<td>-0.96</td>
<td>0.44</td>
<td>-0.89</td>
</tr>
<tr>
<td><strong>Sharpe Ratio</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta</td>
<td>1.79 (5.15)</td>
<td>3.82 (8.3)</td>
<td>24.4 (9.13)</td>
<td>1.81 (14.9)</td>
<td>9.1 (16.05)</td>
<td>-29.76 (6.81)</td>
<td>189.48 (24.08)</td>
</tr>
<tr>
<td>r2</td>
<td>0.31</td>
<td>0.54</td>
<td>0.59</td>
<td>0.79</td>
<td>0.82</td>
<td>0.44</td>
<td>0.91</td>
</tr>
<tr>
<td>Rank Corr.</td>
<td>0.68</td>
<td>0.86</td>
<td>0.85</td>
<td>0.93</td>
<td>0.90</td>
<td>-0.61</td>
<td>0.95</td>
</tr>
</tbody>
</table>

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applied to funds of different sizes, such as a smart beta index that is to become the basis of multiple investment products, ranging from a small segregated account to an exchange-traded fund whose total assets are constantly changing.

We will augment the existing minimum variance strategy with constraints of the form:

$$\sum_{i \in S} \frac{w_i}{ADV} \leq \gamma \sum_{j \in S} \frac{b_j}{ADV}$$  \hspace{1cm} (3)

In other words, we require the weighted average days-to-trade for a given subset of stocks $S$ be equal to or less than that of the same stocks when held at benchmark weights $b$, scaled by some multiple $\gamma$. If $\gamma = 1$, we want the holdings in $S$ to be, on average, at least as liquid as the corresponding holdings in the benchmark. In an abstract sense, overweighting of some illiquid names must be “offset” by underweighting other illiquid names to avoid an across-the-board illiquidity build-up. Comparing with Equation (2), notice that Equation (3) is invariant to both the notional value $N$ and participation rate $c$ as these are present on both sides of the inequality and hence cancel out.

This solution assumes that the benchmark itself is “sufficiently” liquid or at least implementable in some sense. Without going into a detailed appraisal of benchmark construction methodologies, there is indeed widespread consensus among practitioners that most capitalization-weighted benchmark indexes (at least those covering large and mid-cap stocks) are reasonably liquid and investable for all but the largest of fund sizes. Additionally, in most markets, there is a strong relationship between size and trading activity. The burden rests with the index vendor to provide investable compositions.

Care must be taken in selecting the subsets $S$ on which we impose the constraint in Equation (3). Broad buckets would mean mixing stocks with very different liquidity profiles; the constraint therefore can easily be satisfied by holding very large amounts of a very liquid name (e.g., Apple) while maintaining the tilt on illiquid stocks, which is clearly undesirable. In contrast, buckets containing too few stocks would mean very tight constraints, forcing positions back towards benchmark weights. We want to retain flexibility in the strategy; allowing the optimizer to get as close to the minimum variance portfolio as possible, unencumbered.

Determining the appropriate $\gamma$ parameter also requires some experimentation. A value such as $\gamma = 10$ may seem large; recall, however, that an individual position’s contribution to the portfolio’s weighted average days-to-trade is its weight multiplied by its days-to-trade. Multiplying a position’s weight by 3, therefore, actually increases its contribution to portfolio weighted average days-to-trade by nine times. Therefore, what may seem like a very large $\gamma$ does not in fact allow for massive over-weighting of illiquid or small positions relative to their benchmark allocations. We want the minimum variance to preserve its tilt on illiquid stocks, but only slightly so. On the other hand, a low parameter value ($\gamma = 1$, for example) constitutes a very stringent condition to meet, often requiring considerable turnover and “reshuffling” of portfolio weights at rebalance time.

Exhibit 8 presents performance summary statistics for our minimum variance strategy applied to various markets after these liquidity-boosting measures have been implemented, and Exhibit 7 compares their weighted average days-to-trade with the original strategies first shown in Exhibit 3. In every case, returns are reduced slightly compared with the original (and arguably uninvestable) strategy but tradability has been improved significantly; risk-adjusted returns remain very attractive.

Finally, we focus on the case of the U.S. market for a more detailed analysis of the minimum variance portfolio’s liquidity profile, before and after the liquidity constraints are incorporated. Using the Goldman Sachs Shortfall Model, we compute expected transaction costs for trading $500M USD worth of minimum variance compositions spread out evenly over one week, and decompose these cost estimates down to asset-level contributions grouped by liquidity layer. Exhibits 9–11 allow us to visualize the extent of the improvement in tradability. It is easy to see that the most illiquid positions (those requiring weeks or even months to trade) have been eliminated, and the bulk of the portfolio is made up of positions that take less than one day, at most two, to trade. Per Exhibit 9, the liquidity-enhancing measures have the least impact on the USA minimum variance portfolios, owing to the market’s existing high level of liquidity. One could speculate, therefore, that the reduction in transaction costs would be far more significant for minimum variance portfolios in the other global markets.
EXHIBIT 6
Relationship Between Liquidity (Weighted Average Days-to-Trade, Horizontal Axis) and Total Returns, Realized Risk, and Sharpe Ratio (Vertical Axes)

Backtest from January 2002 through May 2014; figures are annualized and denominated in local currency.

(a) Australia, Total Returns
(b) Australia, Realized Risk
(c) Australia, Sharpe Ratio
(d) Canada, Total Returns
(e) Canada, Realized Risk
(f) Canada, Sharpe Ratio
(g) Eurozone, Total Returns
(h) Eurozone, Realized Risk
(i) Eurozone, Sharpe Ratio
(j) Hong Kong, Total Returns
(k) Hong Kong, Realized Risk
(l) Hong Kong, Sharpe Ratio
(m) Japan, Total Returns
(n) Japan, Realized Risk
(o) Japan, Sharpe Ratio
**EXHIBIT 6 (Continued)**

![Graphs showing total returns, realized risk, and Sharpe ratio for U.K. and USA minimum variance portfolios.](image)

**EXHIBIT 7**

Weighted Average Days-to-Trade of Minimum Variance Portfolios, Median Values from 2002–2014

Assumes $500M USD notional and 20% participation rate.

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Canada</th>
<th>Eurozone</th>
<th>Hong Kong</th>
<th>Japan</th>
<th>U.K.</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Variance (original)</td>
<td>101.91</td>
<td>60.93</td>
<td>10.35</td>
<td>351.04</td>
<td>32.29</td>
<td>11.93</td>
<td>3.25</td>
</tr>
<tr>
<td>Minimum Variance (tradable)</td>
<td>12.07</td>
<td>9.43</td>
<td>1.96</td>
<td>20.75</td>
<td>5.10</td>
<td>2.81</td>
<td>1.27</td>
</tr>
<tr>
<td>Days-to-Trade Reduction</td>
<td>88.16%</td>
<td>84.52%</td>
<td>81.05%</td>
<td>94.09%</td>
<td>84.20%</td>
<td>76.41%</td>
<td>60.88%</td>
</tr>
</tbody>
</table>

**FINDINGS**

This brief exercise began by showing how minimum variance strategies, if left relatively “unconstrained,” exhibit heavy biases towards small and illiquid stocks. At the same time, minimum variance backtests in academic research and product marketing collateral often claim vastly superior returns and risk reduction. Upon analyzing these portfolios in terms of tradability and liquidity, the results in the Measuring Tradability section demonstrate that such performance figures may in fact be unattainable in a realistic portfolio management setting. In the section on Liquidity versus Performance, we looked at the performance of minimum variance strategies with different liquidity profiles in several developed markets and found strong empirical evidence that, once tradability is increased (or at least brought to realistic levels), the backtested/theoretical returns of minimum variance strategies almost always declines slightly, in both total and risk-adjusted terms. Minimum variance remains a strong investment strategy—it is simple and transparent, requires few inputs, and often delivers strong returns—but practitioners need to be careful in ensuring liquid, implementable portfolios.

Whether this phenomenon constitutes a liquidity premium embedded in the returns of minimum variance portfolios is debatable. Depending on one’s toler-
Conventional measures aimed in boosting liquidity, such as exclusion or capping the weights of less-liquid names, are likely to degrade minimum variance performance. Minimum variance derives part of its performance from this illiquidity tilt, and a good strategy ought to strike a “sweet spot” between tradability and performance. In the Enforcing Liquidity section, we...
proposed a novel approach to control the weighted average days-to-trade of a portfolio by means of a convex optimization constraint. While this results in marginal performance deterioration, returns remain very attractive and the tradability of the portfolio improves by an order of magnitude. Best of all, the methodology is invariant to portfolio size, making it ideal for “smart beta” index products, which can be applied to funds of different sizes without customization.

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