1 Introduction

Portfolio stress testing provides a means to quantify how a portfolio would perform under extreme economic conditions and identifies a portfolio’s vulnerability to particular scenarios or market events. The two most common types of stress testing used in investment management are (1) historical stress testing and (2) shocking a factor. Historical stress testing analyzes the portfolio under the conditions of a past extreme economic event. Examples of these events include the Asian Currency Crisis in 1997, the Russian Bond Default in 1998, the September 11th Terrorist Attack in 2001, and the Financial Crisis of 2008. Shocking a factor is a stress test that considers how a portfolio would respond to a future hypothetical situation where some aspect of the market or economy is impacted dramatically. Typically, this includes simulating a dramatic change in an economic indicator or the price of a commodity.

In this document, we describe how stress testing results are computed within Axioma Portfolio Analytics. In general, factor risk models are used for stress testing analysis to compute the impact on portfolio returns, in a manner similar to the approach used in performance attribution. The primary difference in the two is that in performance attribution the portfolio holdings and factor returns are for the same period of time whereas in stress testing, the current portfolio holdings are used with factor returns from either a past period of time (historical stress testing) or that represent a hypothetical future scenario (shock a factor).

2 Historical Stress Testing

In considering historical stress testing, let $h_r$ be a current portfolio and suppose that we want to consider how the current portfolio would have performed during a historical period,
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The most obvious way of conducting stress testing is to simply apply the historical asset returns to the current portfolio. But, there are two problems with this. First, the assets may not have existed during the historical period and second, even if they did, any given company may have looked notably different than it does today. So, when we stress test a portfolio, we consider the factor exposures of the portfolio today and consider the returns of those factors from the historical period. This is more in line with what we hope to gain from the stress testing as well. Stress testing is used to consider systemic events, not asset-specific events.

To describe our approach to historical stress testing, we start with defining the returns model. Consider the following returns model:

\[ r_t = X_{t-1}f_t + \varepsilon_t, \]  

(1)

where \( r_t \) is a vector of asset returns, \( X_t \) is a matrix of asset loadings to factors, \( f_t \) is a vector of factor returns, and \( \varepsilon_t \) is a vector of residual, or asset-specific, returns for time period \( t \). Let the \( k \)-th column of \( X_t \) be denoted by \( x_{kt} \) and let \( h \) denote the portfolio of interest. Then, in attributing the return of the portfolio to particular factors in period \( t \), we consider the following decomposition using the returns model given in (1):

\[ r_T^T h = \sum_k f_{kt} x_{kt}^T h + \varepsilon_T^T h, \]  

(2)

where \( f_{kt} x_{kt}^T h \) is the portion of return attributed to factor \( k \) in period \( t \).

The returns during the historical period \( t \) can be computed as

\[ r_T^T h_t = \sum_k f_{kt} x_{kt}^T h_t + \varepsilon_T^T h_t. \]  

(3)

Note that the exposures used in (3) are calculated using the exposure matrix \( X_t \), that is, from the current time \( t \) where the factor returns are as of time \( t \), the historical period. The specific portion of the portfolio return, though, is defined as using the specific asset returns of the historical period. This has the same problem as noted previously that the assets in \( h_t \) may not have existed or may have been notably different at time \( t \). This is much less of an issue in this case though, because the asset-specific returns are generally smaller in magnitude than factor returns and stress testing is, by definition, used to analyze what would happen during certain systemic events. Asset specific events such as the accounting scandal at Enron cannot be captured by stress testing, at least not as described here. For these reasons, we do not consider asset-specific returns in the proposed form of stress testing and \( \varepsilon_t \) is set to zero.

Sample results using this approach are displayed in Figure 1. These samples represent a US Large Cap Index, US Small Cap Index and Japan Index, respectively. The US-centric events have a larger impact on the US indexes whereas the Japan and Asia events have a larger impact on the Japan Index, as would be expected. The magnitude of these returns is similar to those that occurred in these markets at those times. The date ranges for each of these events matches the length of the event or the length of the impact on the market, and the
return is the cumulative return over the entire period. For example, the -20% return for the Japan Index for the Japan Earthquake / Tsunami event is for a 5-day period and the -6% return for the Russian Default and LTCM Crisis is a return over 4 months.

Within the Axioma Portfolio Analytics platform, historical events can be as short as 1 day or can be many weeks or months. Users can use pre-defined events or can define their own events and time ranges for these events. Users can also compare their portfolios side by side with benchmarks or indexes to assess the impact on a more relative basis. Note that results can also vary by the risk model used as well.

3 Shocking a factor

Up to this point, we have described how to perform stress testing over historical periods of interest. Portfolio managers may also want to stress test hypothetical scenarios defined by a significant return to a particular factor.

If the factor of interest is in the original returns model, then the stress testing can be performed as described above where the actual factor returns are replaced by those generated by the user scenario. For example, suppose we want to shock factor \( i \) so that its return is \( \tilde{f}_i \). Using the factor covariance matrix \( \Omega \), we can determine a linear approximation of the impact of the shock on the other factors:

\[
\tilde{f}_j = \Omega_{ij} \frac{\tilde{f}_i}{\Omega_{ii}}.
\]

(4)

We can multiply the vector of estimated factor returns \( \tilde{f} \) by the factor exposure matrix \( X \) and the holdings vector \( h \) to compute the resulting estimated portfolio return \( R \).

\[
R = \sum_k \tilde{f}_k x_k^T h.
\]

(5)

This return can also be decomposed at the factor and asset level.

Figure 2 are sample results for shocking factors in a fundamental risk model.

The sample is for a fictitious US Fundamental Manager. Each of these results is a separate scenario showing the overall impact on the portfolio from shocking one factor. For the sample shown, it can be seen that in some cases the portfolio is less impacted than in other cases. The shock is instantaneous and, therefore, not time horizon dependent.

In the Axioma Portfolio Analytics platform, users can select any risk factor and shock the factor in a positive or negative amount. For risk models with a market factor, the market factor can be shocked to simulate the impact of shifts in the global, regional or local markets.
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(a) Results for a US Large Cap Index
(b) Results for a US Small Cap Index
(c) Results for a Japan Index

Figure 1: Sample Historical Event Stress Testing Results
For shocking factors, a more typical use case might be to shock economic indicator factors such as oil prices and credit spreads from a macroeconomic risk model instead of factors in a fundamental risk model. The Axioma Portfolio Analytics platform will include these types of factors as part of a US macroeconomic model which is scheduled for release at the end of 2013.

Lastly, when shocking risk model factors, the nature of the factor is important to keep in mind when selecting the magnitude of the shock or interpreting the results. In some cases, factor returns might be excess returns relative to a market or other factor. In other cases, the typical magnitude of the factor returns might determine the appropriate scale of a shock and define if the shock is large or small. For example, industry factors may have daily factor returns on the order of -5% to +5% but style factor returns may be much smaller and on the order of -0.5% to +0.5%.

4 Conclusion

This document presented the algorithms used for stress testing analysis on the Axioma Portfolio Analytics platform. These include both historical stress testing and shocking a risk model factor. In both cases, the impact on the portfolio’s performance is estimated. These analyses complement the other risk analysis that are part of this product. Stress testing can be performed using any of the Axioma fundamental equity or macroeconomic factor risk models. Custom risk models (generated using the Axioma Risk Model Machine) can also be used. Statistical risk models should not be used in this type of stress testing framework.
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